EE3123 Tutorial 3 (Solution)
Three-phase power circuit analysis

Name: **Student No.:**

Q1

A Y-connected balanced three-phase 240-V source supplies a balanced three-phase load. If the line current I_A is measured to be 15 A and is in phase with the line-to-line voltage V_{BC}, find the per-phase load impedance if the load is (a) Y-connected, (b) Δ-connected.

Solution

(a) $\bar{V}_{AN} = \frac{240}{\sqrt{3}}\angle 0° = 138.56\angle 0°\,\text{V}$ (Assumed as Reference)

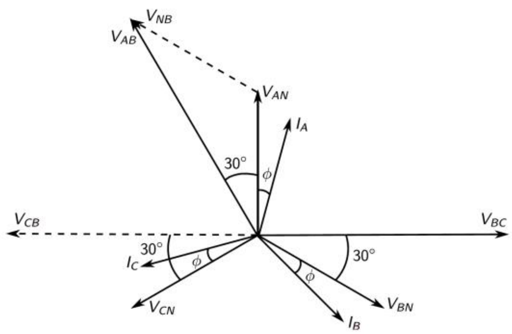
$$\bar{V}_{AB} = 240\angle 30°\,\text{V};\ \bar{V}_{BC} = 240\angle -90°\,\text{V};\ \bar{I}_A = 15\angle -90°\,\text{A}$$

$$\bar{Z}_Y = \frac{\bar{V}_{AN}}{\bar{I}_A} = \frac{138.56\angle 0°}{15\angle -90°} = 9.24\angle 90° = (0 + j9.24)\,\Omega$$

(b) $\bar{I}_{AB} = \frac{\bar{I}_A}{\sqrt{3}}\angle 30° = \frac{15}{\sqrt{3}}\angle -90° + 30° = 8.66\angle -60°\text{A}$

$$\bar{Z}_\Delta = \frac{\bar{V}_{AB}}{\bar{I}_{AB}} = \frac{240\angle 30°}{8.66\angle -60°} = 27.71\angle 90° = (0 + j27.71)\,\Omega$$

Note: $\bar{Z}_Y = \bar{Z}_\Delta / 3$



Q2

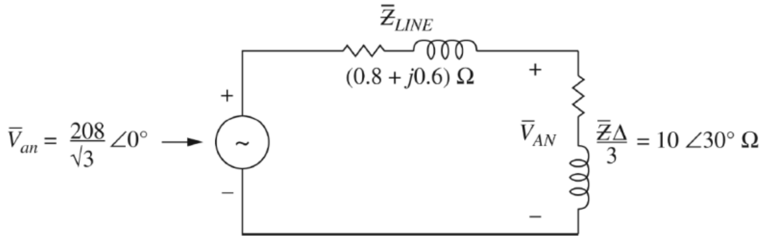
Three identical impedances $Z_\Delta = 30\angle 30°\ \Omega$ are connected in Δ to a balanced three-phase 208-V source by three identical line conductors with impedance $Z_L = (0.8 + j0.6)\ \Omega$ per line.

(a) Calculate the line-to-line voltage at the load terminals.

(b) Repeat part (a) when a Δ-connected capacitor bank with reactance ($-j60$) Ω per phase is connected in parallel with the load.

Solution

(a)



Using voltage division: $\overline{V}_{AN} = \overline{V}_{an} \dfrac{\overline{Z}_{\Delta}/3}{(\overline{Z}_{\Delta}/3) + \overline{Z}_{LINE}}$

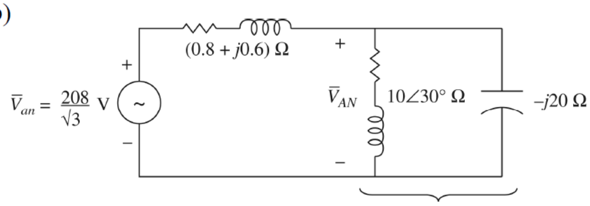
$$= \frac{208}{\sqrt{3}}\angle 0° \frac{10\angle 30°}{10\angle 30° + (0.8 + j0.6)}$$

$$= \frac{(120.09)(10\angle 30°)}{9.46 + j5.6} = \frac{1200.9\angle 30°}{10.99\angle 30.62°}$$

$$= 109.3\angle -0.62° \text{ V}$$

Load voltage = $V_{AB} = \sqrt{3}\,(109.3) = 189.3\,\text{V}$ Line-to-Line

(b)



$$\overline{Z}_{eq} = 10\angle 30° \,||\, (-j20)$$
$$= 11.547\angle 0° \Omega$$

$$\overline{V}_{AN} = \overline{V}_{an} \frac{\overline{Z}_{eq}}{\overline{Z}_{eq} + \overline{Z}_{LINE}}$$

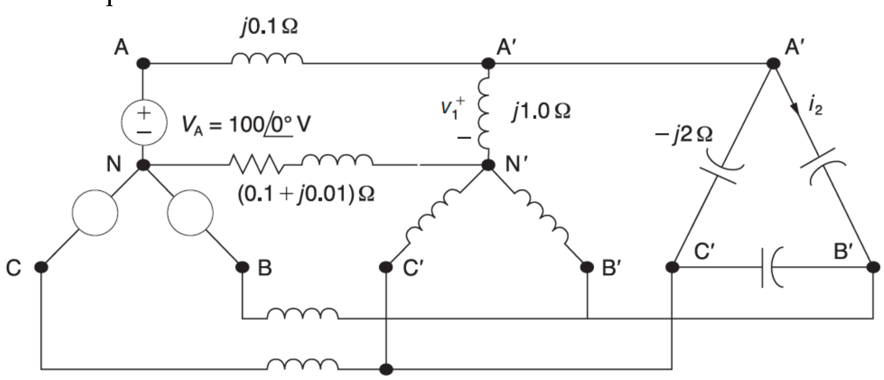
$$= \left(208/\sqrt{3}\right)\frac{11.547}{(11.547 + 0.8 + j0.6)}$$

$$= \frac{1386.7}{12.362\angle 2.78°} = 112.2\angle -2.78° \text{ V}$$

Load voltage Line-to-Line $V_{AB} = \sqrt{3}\,(112.2) = 194.3$ V

Q3

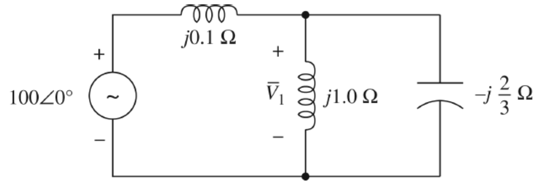
Consider the balanced three-phase system shown below. Determine $v_1(t)$ and $i_2(t)$. Assume positive phase sequence.



Solution

Replace delta by the equivalent WYE: $\bar{Z}_Y = -j\frac{2}{3}\,\Omega$

Per-phase equivalent circuit is shown below:



Noting that $\left(j1.0 \,\middle\|\, -j\frac{2}{3}\right) = -j2$, by voltage-divider law,

$$\bar{V}_1 = \frac{-j2}{-j2 + j0.1}\left(100\angle 0°\right) = 105\angle 0°$$

$$\therefore \upsilon_1(t) = 105\sqrt{2}\cos(\omega t + 0°) = 148.5\cos\omega t \text{ V} \quad \leftarrow$$

In order to find $i_2(t)$ in the original circuit, let us calculate $\bar{V}_{A'B'}$

$$\bar{V}_{A'B'} = \bar{V}_{A'N'} - \bar{V}_{B'N'} = \sqrt{3}\,e^{j30°}\bar{V}_{A'N'} = 181.9\angle 30°$$

Then $$\bar{I}_{A'B'} = \frac{181.9\angle 30°}{-j2} = 86.6\angle 120°$$

$$\therefore i_2(t) = 90.9\sqrt{2}\cos\left(\omega t + 120°\right)$$

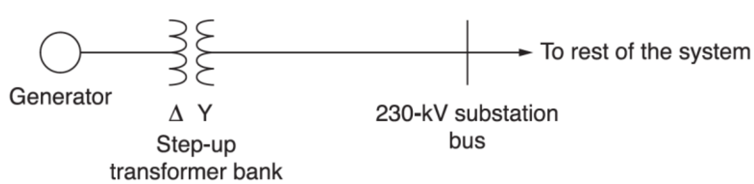
$$= 128.6\cos\left(\omega t + 120°\right)\text{A} \quad \leftarrow$$

Q4

Consider the one-line diagram shown below. The three-phase transformer bank is made up of three identical single-phase transformers, each specified by X_l=0.24 Ω (on the low-voltage side), negligible resistance and magnetizing current, and turns ratio $n=N_2/N_1=10$. The transformer bank is delivering 100 MW at 0.8 p.f. lagging to a substation bus whose voltage is 230 kV (line voltage).

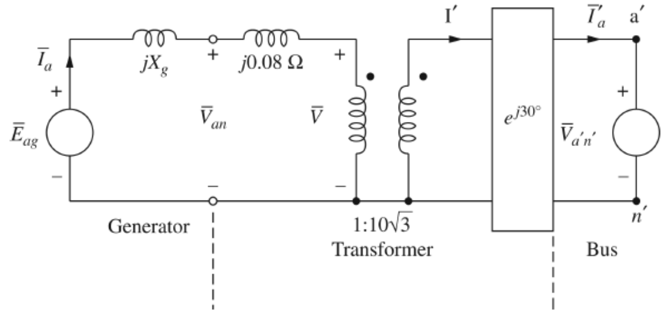
(a) Determine the primary current magnitude, primary voltage (line-to-line) magnitude, and the three-phase complex power supplied by the generator. Choose the line-to-neutral voltage at the bus, $V_{a'n'}$ as the reference. Account for the phase shift, and assume positive-sequence operation.

(b) Find the phase shift between the primary and secondary voltages.



Solution

(a) For positive sequence operation and standard Δ-Y connection, the per-phase diagram is shown below:



$$\overline{V}_{a'n'} = \frac{230}{\sqrt{3}}\angle 0°\,\text{kV}, \text{ choosing that as a reference.}$$

$$\overline{S}' = \frac{100\times 10^6}{0.8\times 3}\angle \cos^{-1} 0.8 = 41.67\angle 36.87°\,\text{MVA}$$

$$\overline{I}'^{*}_a = \frac{\overline{S}'}{\overline{V}_{a'n'}} = \frac{41.67\times 10^6\angle 36.87°}{132.8\times 10^3} = 313.8\angle 36.87°\,\text{A}$$

$$\therefore \overline{I}'_a = 313.8\angle -36.87°\,\text{A}$$

$$\overline{I}_a = 10\sqrt{3}\,e^{-j30°}\overline{I}'_a = 5435\angle -66.87°\,\text{A}$$

The primary current magnitude is 5435A. ←

$$\overline{V}_{an} = \overline{V} + j0.08\overline{I}_a = \left(\frac{1}{10\sqrt{3}}132.8\times 10^3\angle -30°\right) + j0.08(5435\angle -66.87°)$$

$$= 7667.4\angle -30° + 434.8\angle 23.13°$$

$$= 7935.9\angle -27.49°\,\text{V}$$

Line-to-line primary voltage magnitude $= \sqrt{3}(7935.9) = 13.75\,\text{kV}$ ←

Three-phase complex power supplied by the generator is

$$\overline{S}_{3\phi} = 3\overline{V}_{an}\overline{I}_a^{*} = 3(7935.9\angle -27.49°)(5435\angle 66.87°)$$

$$= 129.39\angle 39.38°\,\text{MVA} \leftarrow$$

(b) The secondary phase leads the primary by 27.49°; this phase shift applies to line-to-neutral (phase) as well as line-to-line voltages. ←